V Semester B.A./B.Sc. Examination, March/April 2022
(Semester Scheme)
(CBCS) (F + R) (2016 – 17 and Onwards)
MATHEMATICS (Paper – V)

Time : 3 Hours | Max. Marks : 70

***Instruction** : Answer **all** questions.*

PART – A

1. Answer **any five** questions. (5×2=10)

a) In a ring (R, +, .), show that $a\cdot(-b) = (-a)\cdot b = -(a\cdot b)$ for all $a, b \in R$.

b) Define subring of a ring. Give an example.

c) Give an example of
   i) Commutative ring without unity.
   ii) A non commutative ring without unity.

d) If $\phi(x, y, z) = x^2y^2z^2$ and $\vec{F} = 2x\hat{i} + y\hat{j} + 3z\hat{k}$ find $\vec{F}.\nabla\phi$.

e) Find the unit normal vector to the surface $x^2 - y^2 + z = 3$ at the point (1, 0, 2).

f) Evaluate : $\Delta^3[(1 - x)(1 - 2x)(1 - 3x)]$.

g) Write Lagrange's interpolation formula.

h) Evaluate : $\int_0^1 \frac{dx}{1+x}$ using Trapezoidal rule, given

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

PART – B

Answer **two full** questions. (2×10=20)

2. a) Prove that every field is an integral domain.
   Is the converse of the above theorem is true ? Justify with example.

   b) Show that set R = {0, 1, 2, 3, 4, 5} is a commutative ring w.r.t $\oplus_6$ and $\otimes_6$ as two compositions.

OR

3. a) Prove that a ring R without zero divisors if and only if the cancellation laws holds.

   b) Show that necessary and sufficient condition for a non-empty subset S of a ring R to be a subring of R are

      i) $a - b \in S \quad \forall a, b \in S$

      ii) $ab \in S \quad \forall a, b \in S$.

4. a) Prove that an ideal S of the ring (z, +,·) is maximal if and only if S is generated by some prime integer.

   b) Find all the principal ideals of the ring R = {0, 1, 2, 3, 4, 5} w.r.t $\oplus_6$ and $\otimes_6$

OR

5. a) If $f : R \to R'$ be a homomorphism of R into R′, then show that Ker f is an ideal of R.

   b) State and prove fundamental theorem of homomorphism.

PART – C

Answer **two full** questions. **(2×10=20)**

6. a) Find the directional derivative of $\phi(x, y, z) = xyz - xy^2z^3$ at the point (1, 2, –1) in the direction of $\hat{i} - \hat{j} - 3\hat{k}$.

   b) If $\vec{F} = \text{grad}(x^3 + y^3 + z^3 - 3xyz)$, find div $\vec{F}$ and curl $\vec{F}$.

OR

7. a) Find the values of 'a' and 'b' so that the surface $5x^2 - 2yz - 9x = 0$ may cut the surface $ax^2 + by^3 = 4$ orthogonally at (1, – 1, 2).

   b) If ϕ is a scalar point function and $\vec{F}$ is a vector point function, prove that div $(\phi\vec{F}) = \phi(\text{div}\vec{F}) + (\text{grad}\,\phi).\vec{F}$.

8. a) If $\vec{u} = x^2\,\hat{i} + y^2\,\hat{j} + z^2\hat{k}$ and $\vec{v} = yz\,\hat{i} + zx\,\hat{j} + xy\hat{k}$. Show that $\vec{u} \times \vec{v}$ is a solenoidal vector.

   b) Show that $\vec{F} = (6xy + z^3)\,\hat{i} + (3x^2 - z)\,\hat{j} + (3xz^2 - y)\,\hat{k}$ is irrotational. Find ϕ such that $\vec{F} = \nabla\phi$.

OR

9. a) Prove that :

      i) div(curl $\vec{F}$) = 0.

      ii) curl (grad ϕ) = 0.

   b) For any vector field $\vec{f}$ and $\vec{g}$ prove that $\text{div}(\vec{f} \times \vec{g}) = \vec{g}.\text{curl}\,\vec{f} - \vec{f}.\text{curl}\,\vec{g}$.

PART – D

Answer **any two full** questions. **(2×10=20)**

10. a) Use the method of separation of symbols. Prove that

$$u_0 + \frac{u_1 x}{1!} + \frac{u_2 x^2}{2!} + ... = e^x \left[u_0 + x\frac{\Delta u_0}{1!} + x^2 \frac{\Delta u_0}{2!} + ... \right].$$

b) Obtain the function whose first difference is $6x^2 + 10x + 11$.

OR

11. a) Find a cubic polynomials which takes the following data and hence evaluate f(4).

x	0	1	2	3
f(x)	1	2	1	10

b) Find f(1.4) from the following data using difference table.

x	1	2	3	4	5
f(x)	10	26	58	112	194

12. a) Use Newton divided difference formula and find f(8) from the following data :

x	1	3	6	11
f(x)	4	32	224	1344

b) Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by using Simpson's $\frac{3}{8}^{th}$ rule by taking n = 6.

OR

13. a) By using Lagrange's interpolation formula, find f(6) from the following data :

x	3	7	9	10
f(x)	168	120	72	63

b) Evaluate $\int_0^{0.6} e^{-x^2} dx$ by taking 6 subintervals by using Simpson's $\frac{1}{3}^{rd}$ rule.
